Uniformly Accelerated Motion

Week 3, Lesson 2

- Units of Length & Time
- Speed
- Distance vs. Displacement
- Average Speed vs. Average Velocity
- Instantaneous Velocity
- Acceleration
- Uniformly Accelerated Motion Along a Straight Line

References/Reading Preparation:
Schaum’s Outline Ch. 4
Principles of Physics by Beuche – Ch.2

Uniformly Accelerated Motion

In this situation, we will limit ourselves to situations in which acceleration is constant , or in which an object is uniformly accelerated.

For example:

- objects falling freely under the action of gravity near the earth's surface have constant acceleration.

Units of Length and Time

In order to define quantities which describe motion, we must first define the basic units of measurement of length and time.

The basic SI unit of length is the ***meter.***

The basic SI unit of time is the ***second.***

Speed

When we say that a car is moving at a speed of 80 km/h, everybody knows what we mean:

The car will go a distance of 80 km in 1 hr -

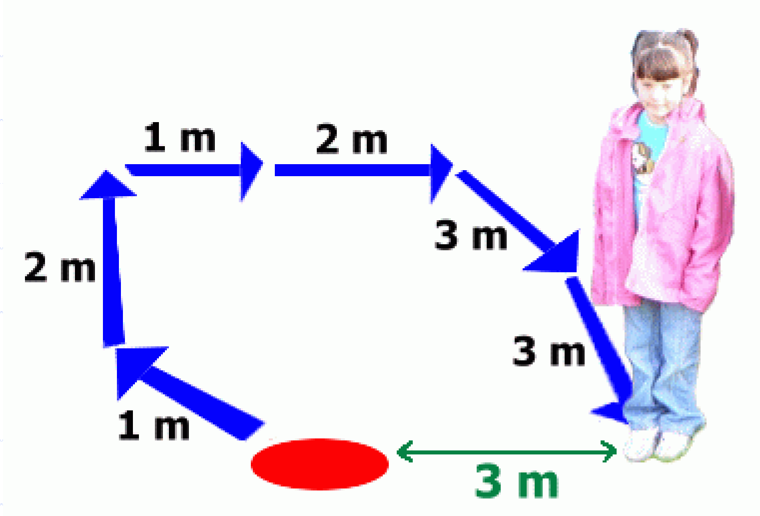
provided it maintains this speed.

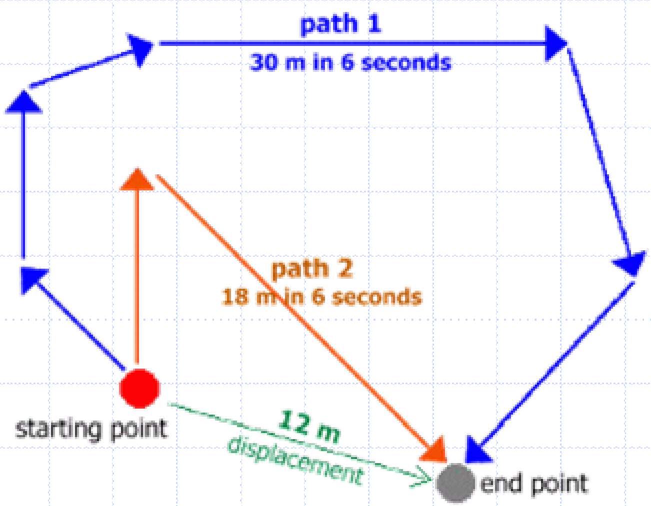
In general, the distance a car travels if its speed is constant is:

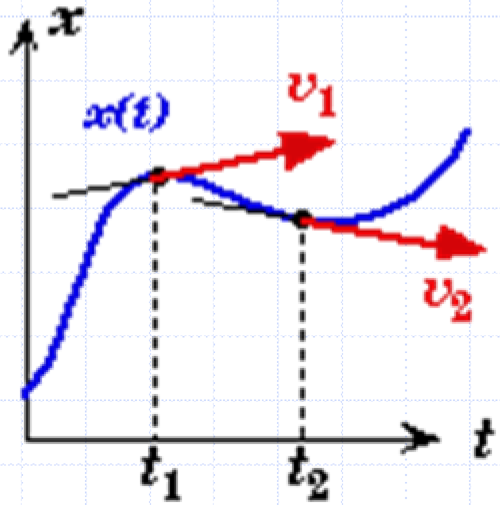
Distance traveled = speed x time taken

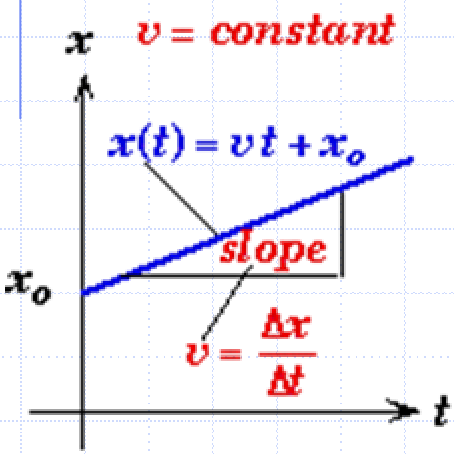
$$\text{Average Speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

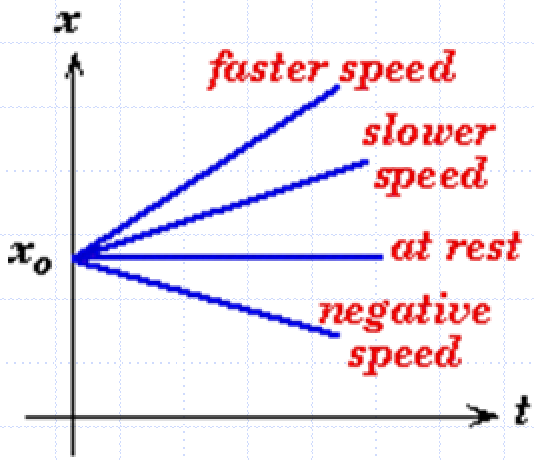
$$\text{Average Velocity} = \frac{\text{final displacement}}{\text{total time taken}}$$

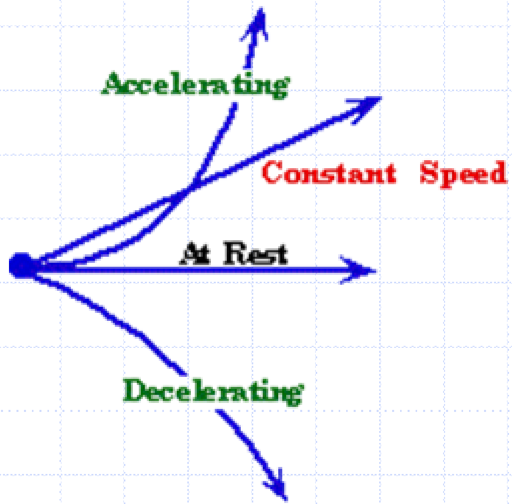





x
x(t)
v_1
v_2
t
t_1
t_2


x
v = constant
x(t) = v t + x_o
slope
x_o
$v = \frac{\Delta x}{\Delta t}$
t


x
faster speed
slower speed
x_o
at rest
negative speed
t


Accelerating
Constant Speed
At Rest
Decelerating

Average Velocity

$$\bar{v} = \frac{x_f - x_o}{t_f - t_o} = \frac{\Delta x}{\Delta t}$$

Instantaneous Velocity

$$v = \lim_{\Delta t \to 0} \bar{v} = \frac{dx}{dt}$$

Speed

Solving for speed:

$$\text{Speed} = \frac{\text{distance traveled}}{\text{time taken}} = \frac{d}{t}$$

We use this same equation to define the average speed of a car when its speed is not constant.

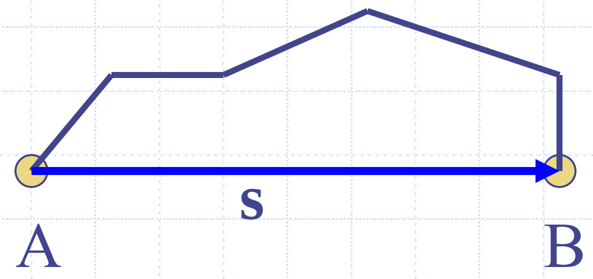
If the car goes 200 km in 4.0 h, its average speed is:

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time taken}}$$

Displacement

Suppose A and B represents two cities.

B is 800 km directly east of A.



There are many roads from A to B – each a different *distance*.

One road is shown.   It is 1200 km long.

The shortest distance is represented by the vector **s**.

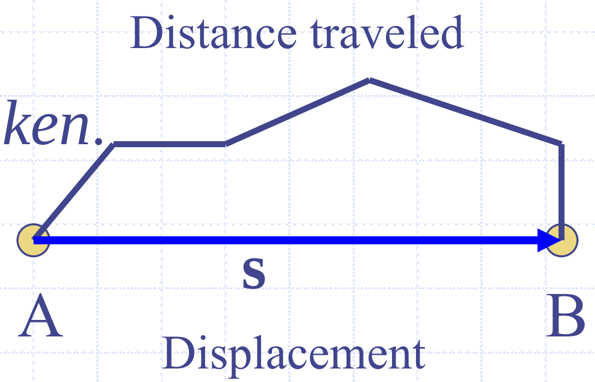
s is called the displacement from A to B.

Distance vs. Displacement

- You can see the difference between distance traveled and displacement.

Distance traveled depends on the *path taken*.

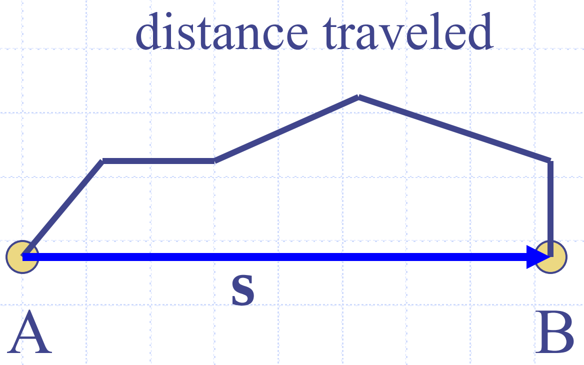
Displacement is independent of path.



Average Speed vs. Average Velocity

- We have seen that

$$\text{Average speed} = \frac{\text{distance traveled}}{\text{time taken}}$$



By definition:

$$\text{Average velocity} = \frac{\text{Displacement vector}}{\text{time taken}}$$

In symbols: $\bar{\mathbf{v}} = \dfrac{\mathbf{s}}{t}$

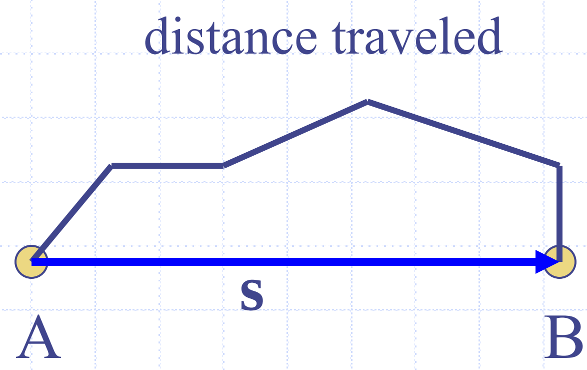
The bar above the **v** is used to indicate average velocity.

Average Speed vs. Average Velocity

Note:

SPEED is a scalar quantity.

VELOCITY is a vector quantity.



If I travel along the path shown (black line) in 20 h, then:

$$\text{My average speed} = \frac{\text{distance traveled}}{\text{time taken}} = \frac{1200\ \text{km}}{20\ \text{h}} = 60\ \text{km/h}$$

BUT

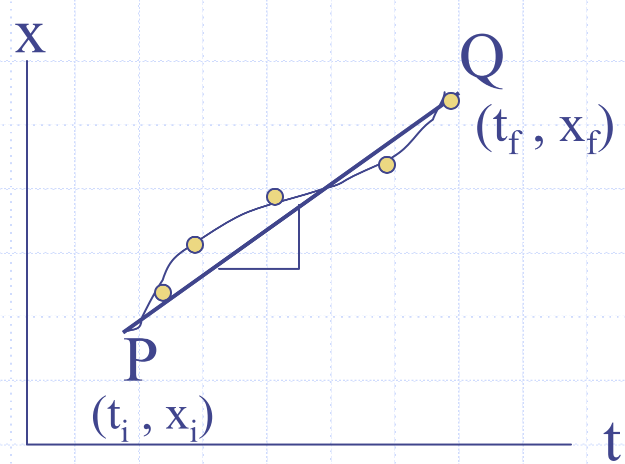
$$\text{My average velocity} = \frac{\text{displacement vector}}{\text{time taken}} = \frac{800\ \text{km}}{20\ \text{h}} = 40\ \text{km/h, east}$$

Instantaneous Velocity

- If we move along the x-axis from:

x_i at t_i → x_f at t_f

We can make a plot like this:



distance $\Delta x = x_f - x_i$

time $\Delta t = t_f - t_i$

The average velocity is the slope of the line joining P and Q.

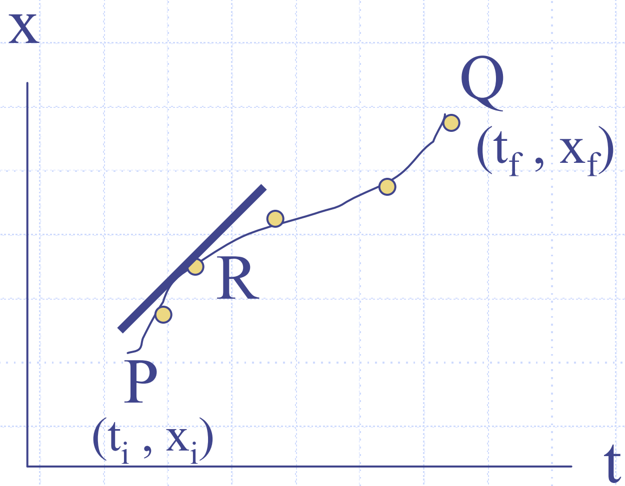
$$\bar{\mathbf{v}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Looking at our plot, let Q approach P, therefore Δt gets smaller, and

$$\text{Instantaneous velocity} = \mathbf{v} = \lim_{\Delta t \to 0} \Delta x / \Delta t$$

Instantaneous Velocity

From this we can see that the ***Instantaneous Velocity*** at a a certain time *t* is the slope of the curve at that time.



Illustration

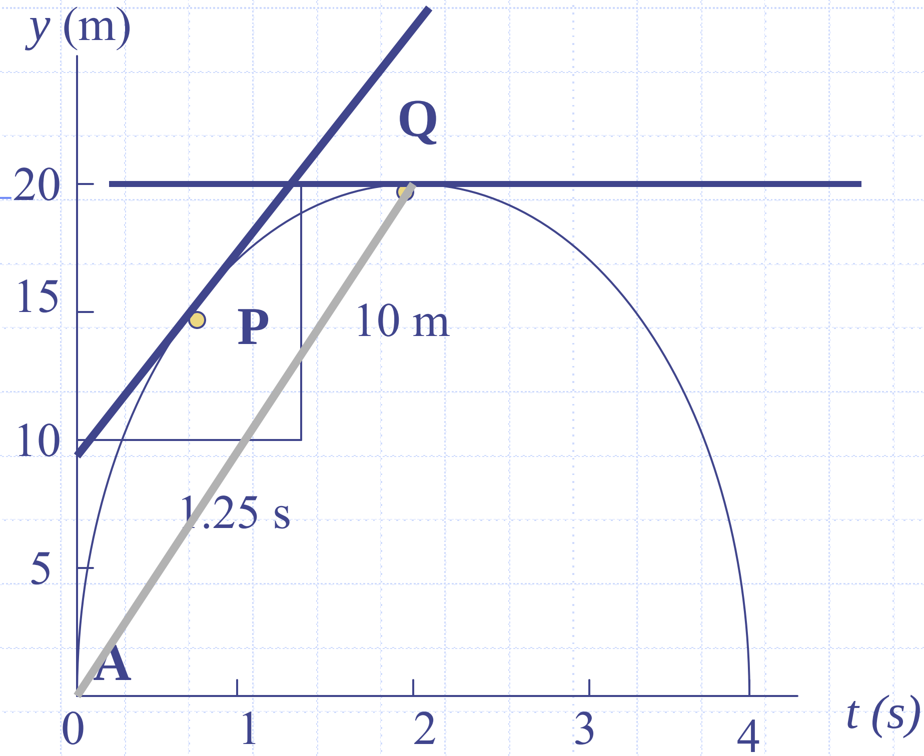
A ball is thrown straight up. (It goes up, then stops, changes direction, then falls to the ground).

The Figure on the following slide shows the ball,s y-coordinate (vertical displacement) as a function of time.

Let us find the ball's instantaneous velocity at points P and Q.

Let us also find the average velocity between points A and Q.

Illustration



Instantaneous Velocity at points **P** and **Q** is the slope of the tangent at each point.

$\mathbf{v}_p$ = slope at P = 10 m/1.25 s = 8 m/s

$\mathbf{v}_Q$ = slope at Q = 0

The average velocity between **A** and **Q** is the slope of the line joining the two: $\overline{\mathbf{v}}_{AQ}$ = 20 m / 2.0 s = 10 m/s

Acceleration:

ACCELERATION measures the time rate change of velocity.

It is defined as:

$$\bar{\mathbf{a}} = \textit{average acceleration} = \frac{\text{change in velocity vector}}{\text{time taken}}$$

$$= \frac{\mathbf{v}_f - \mathbf{v}_o}{\boldsymbol{t}}$$

where: $\mathbf{v}_f$ = *final velocity*
$\mathbf{v}_o$ = *original velocity*
t = *time interval over which the change occurred*

Typical units are (m/s)/s or m/s^2

Uniformly Accelerated Motion Along a Straight Line

In this case, the acceleration vector is constant
and
along the line of the displacement vector.

Because motion is in a straight line, we simplify our discussion by using plus and minus signs to show direction.

(positive if in the positive direction and negative if in the negative direction.)

and, we represent:

1) The vector displacement by *x*.
2) The x-directed velocity by *v*.
3) The x-directed acceleration by *a*.

Uniformly Accelerated Motion – cont'd

The motion can now be described with the following equations:

1) $x = \bar{v}t$

2) $\bar{v} = \dfrac{v_f + v_o}{2}$

3) $a = \dfrac{v_f - v_o}{t}$

4) $v_f^2 = v_o^2 + 2ax$

5) $x = v_o t + \dfrac{at^2}{2}$

Let's do some sample questions.

4.1 (Schaum's)

4.3 (Schaum's)

Uniformly Accelerated Motion – cont'd

Week 4, Lesson 1

- Review
- Acceleration due to Gravity
- Uniformly Accelerated Motion Along a Straight Line

References/Reading Preparation:
Schaum's Outline Ch. 4
Principles of Physics by Beuche – Ch.2

Review of last lesson

We've seen that uniformly accelerated motion along a straight line can now be described with the following equations:

1) $x = \bar{v}t$

2) $\bar{v} = \dfrac{v_f + v_o}{2}$

3) $a = \dfrac{v_f - v_o}{t}$

4) $v_f^2 = v_o^2 + 2ax$

5) $x = v_o t + \dfrac{at^2}{2}$

These equations can apply to two primary situations:

1. Motion along a straight line (say the x-axis).

And,

2. Free falling bodies – in which the body accelerates downward with an acceleration of 9.8 m/s^2.

In this course (General Physics 1) you will be expected to apply these formulae to these two situations.

Acceleration Due to Gravity

Acceleration Due to Gravity (*g*):

The acceleration of a body moving under the force of gravity is g, the gravitational (or free-fall) acceleration, which is directed vertically downward.

On earth, $g = 9.8\ m/s^2$ ($=32.2\ ft/s^2$); which can vary slightly from place to place.

Graphical Interpretations

As we have seen, graphical interpretations for motion along a straight line (the x-axis) are as follows:

- The ***instantaneous velocity*** of an object at a certain time is the slope of the x-versus-t graph at that time.
- The ***instantaneous acceleration*** of an object at a certain time is the slope of the v-versus-t graph at that time.
- For constant-velocity motion, the x-versus-t graph is a straight line.
- For constant-acceleration motion, the v-versus-t graph is a straight line.

Example question – linear motion.

4.10 A bus moving at a speed of 20 m/s begins to slow down at a rate Of 3 m/s each second. Find how far it goes before stopping.

Take the direction of motion to be the +ve *x*-direction.

$v_o = 20$ m/s

$v_f = 0$ m/s

$a = -3$ m/s^2 Note that the bus is slowing down and so the acceleration is negative (deceleration).

Using $v_f^2 = v_o^2 + 2ax$

$$x = \frac{-(20 \text{ m/s})^2}{2(-3 \text{ m/s}^2)} = 66.7 \text{ m}$$

Example – Falling Object

4.8 A ball is dropped from rest at a height of 50 m above the ground.

a) What is its speed before it hits the ground?

b) How long does it take to reach the ground?

Note, for these types of questions, we ignore air friction.

For this question, we take up as positive:

We have: y = -50 m ; a = -9.8 m/s^2 ; v_o = 0

a) Using the formula: $v_f^2 = v_o^2 + 2ay$

$$= 0 + 2(-9.8\text{ m/s}^2)(-50\text{ m}) \quad = 980\text{ m}^2/\text{s}^2$$

$$v_f = \pm\sqrt{980\text{ m}^2/\text{s}^2} \quad = -32.1\text{ m/s}$$

b) From $a = (v_f - v_o)/t$

$$t = \frac{v_f - v_o}{a} = \frac{(-31.3 - 0)\text{ m/s}}{-9.8\text{ m/s}^2} = 3.19\text{ s}$$

Example question – falling object

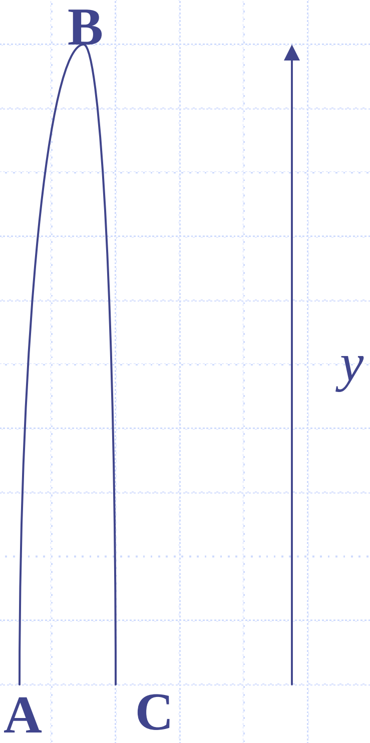
A ball is thrown upward with a velocity of 15 m/s.

a) How high does it go?

b) What is its velocity just before it is caught?

c) How long was it in the air?

a) First, draw a sketch.



We can look at the problem in two parts.

First, the trip from A to B. (up is +ve)

$a = -9.8\ \text{m/s}^2$; $v_o = 15\ \text{m/s}$; $v_f = 0$; $y = ??$

Using the equation: $v_f^2 = v_o^2 + 2ay$ (note change to y)

$y = -(15)^2 / 2(-9.8) = 11.5\ \text{m}$ (from A to B)

For the second part, we are concerned with the trip from B to C.

b) This is a new trip, so we have new knowns:

$a = -9.8 \text{ m/s}^2$; $y = -11.5$ m ; $v_o = 0$; $v_f = ??$

Using $2ay = v_f^2 - v_o^2$

$$v_f = \pm\sqrt{2(-9.8)(-11.5)}$$
$$= -15 \text{ m/s}$$ (-ve because final velocity is downward)

c) To find the length of time in the air, we use the formula:

$y = v_o t + at^2/2$ (The first term = 0 since $v_o = 0$)

From which we get that: $t^2 = 2y/a$

$$= 2(-11.5)/(-9.8) = 2.35 \text{ s}^2$$

$t = 1.53$ s (recall that this is the time from B to C)

Therefore, the TOTAL time in the air is 2 x 1.53s = 3.06 s

Projectile Problems

Projectile problems (problems in which something is thrown or moving through the air) are a combination of an object moving linearly (in a horizontal direction) and falling under the effect of gravity.

Therefore, we consider the object to consist of 2 *independent parts:*

1) A ***horizontal*** component with $a = 0$ and $v_f = v_o = \overline{v} = v_h$

AND

2) A ***vertical*** component with $a = g = 9.8$ m/s^2 downward.

We also ignore air friction.

Example Projectile Problem

Suppose a ball leaves a thrower's hand horizontally with a velocity of 15 m/s from a position 2.0 m above the ground. Where will it strike the ground?

This problem is solved in two parts. Since the horizontal velocity of 15 m/s remains constant throughout the ball's path, the distance it travels depends on the total time it is in the air.

The time it is in the air depends on *g* and the time it takes to drop to the ground.

Therefore, we solve this in two parts:

1) The time it takes to fall to the ground

$$v_0 = 0 \; ; \; a = -9.8 \text{ m/s}^2 \; ; \; y = -2.0 \text{ m}$$

Using: $y = v_o t + \frac{1}{2} a t^2$

$$t = \sqrt{2(-2.0)/(-9.8)}$$

$= 0.639$ s (this is the time it takes to fall to the ground)

2) The horizontal distance it travels in 0.639 s.

$$v_h = 15 \text{ m/s}$$

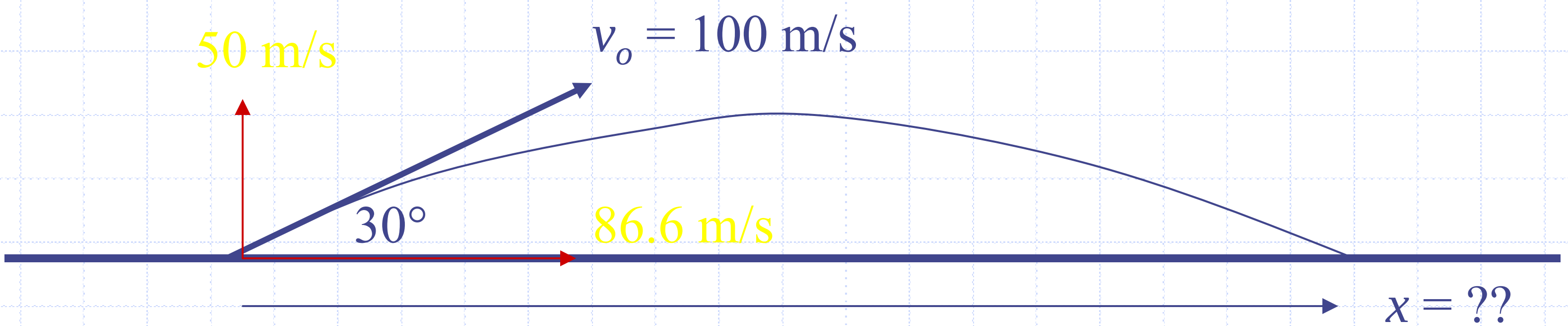
Using $x = v_h(t)$

$$x = 15 \text{ m/s} \times 0.639 \text{ s} = 9.58 \text{ m}$$

It will strike the ground 9.58 m from where it was released.

Projectile Fired at an Angle

4.20 A ball is thrown with an initial velocity of 100 m/s at an angle of 30° above the horizontal, as shown. How far will the ball travel assuming that it will hit the ground at the same elevation it was thrown at?



To solve this, we must divide the problem into horizontal and vertical parts.

Therefore, $v_{ox} = v_o \cos 30° = 86.6$ m/s

And $v_{oy} = v_o \sin 30° = 50$ m/s

In the vertical problem, $y = 0$ since the ball returns to its original height. Then:

$$y = v_{oy}t + ½\, a_y t^2$$

or

$$0 = (50 \text{ m/s}) + ½(-9.8 \text{ m/s}^2)t$$

And $t = 10.2$ s

In the horizontal problem, $v_{ox} = v_{fx} = 86.6$ m/s

Therefore:

$$x = v_{fx}t$$

$$= (86.6 \text{ m/s})(10.2 \text{ s})$$

$$= 884 \text{ m}$$

EXAMPLES

1) An airplane accelerates down a run-way at 3.20 m/s^2 for 32.8 s until is finally lifts off the ground. Determine the distance traveled before takeoff.

ANS: d = 1720 m

2) A car starts from rest and accelerates uniformly over a time of 5.21 seconds for a distance of 110 m. Determine the acceleration of the car.

ANS: a = 8.10 m/s^2

3) Chuck is riding the Giant Drop at Great America. If Chuck free falls for 2.6 seconds, what will be his final velocity and how far will he fall?

ANS: d = 33 m; v_f = -25.5 m/s

4) A race car accelerates uniformly from 18.5 m/s to 46.1 m/s in 2.47 seconds. Determine the acceleration of the car and the distance traveled.

ANS: a = 11.2 m/s^2 ; d = 79.8 m

5) A car is traveling at a speed of 80 ft/s when the brakes are suddenly applied, causing a constant deceleration of 10 ft/s^2. Determine the time required to stop the car and the distance traveled before stopping.